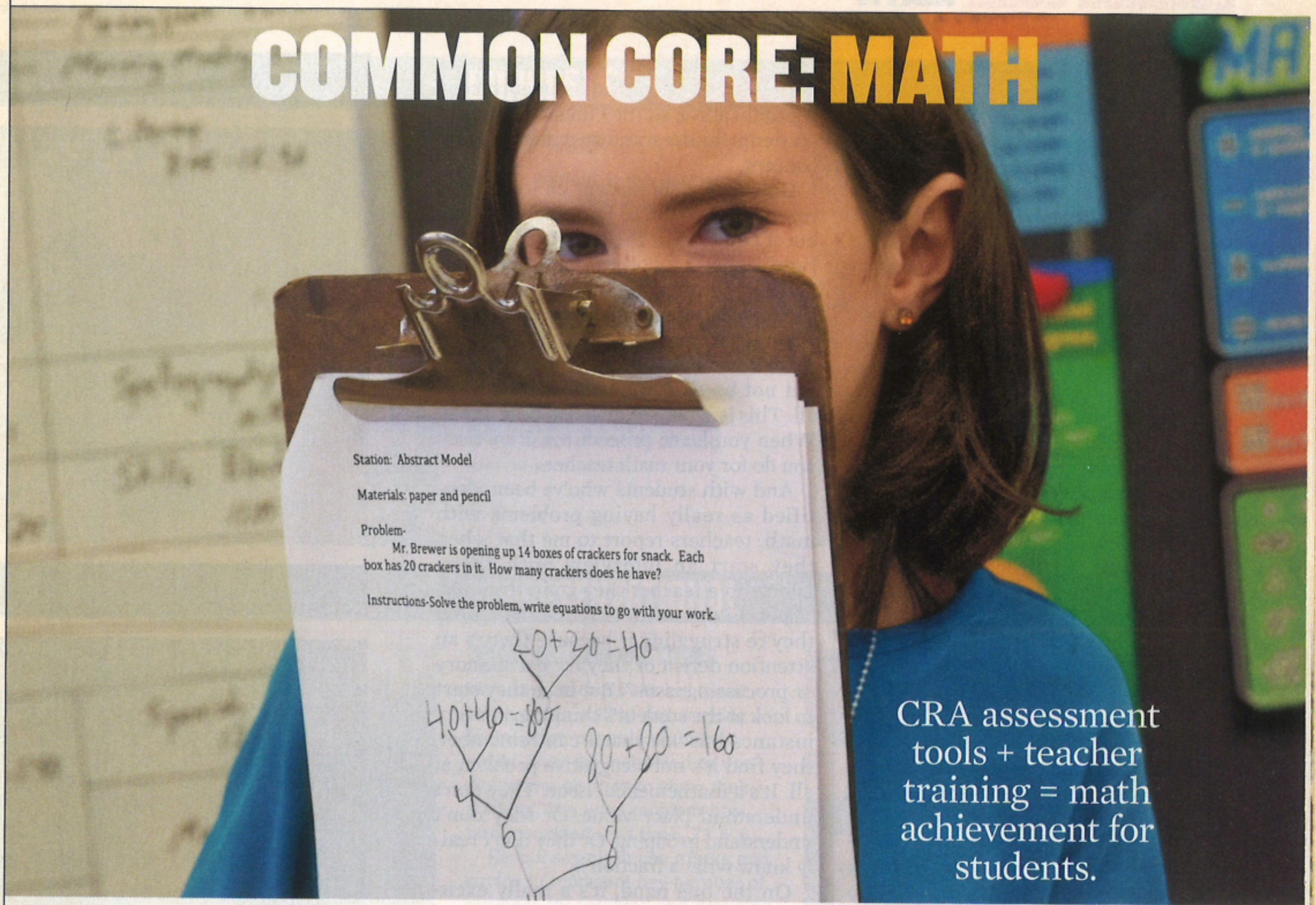


AN EQUATION FOR SUCCESS

THE NEW COMMON CORE STANDARDS are bringing a number of changes, but their impact is being felt most by elementary school teachers. Many of them are experts in literacy. They know enough math to get by, but they aren't necessarily thinking, or teaching, in terms of the fundamental concepts. The Core demands big changes in math instruction, and it's your job to make sure your teachers are up to the task. Education professor John Tapper is ready to help with his new book, *Solving for Why*. He says teachers tend to overemphasize procedures—and getting right answers—when they should move beyond right and wrong to exploring what students understand, where they are struggling, and, most important, why they are struggling.

We talked with Tapper about the concrete-representational-abstract assessments, or CRA, a tool that does just that. It's easy to implement, and it provides a sophisticated portrait of kids' models for mathematical concepts. Here's a handbook for helping your lower-grade teachers, and their students, achieve math success.

BY ERICH STROM

PHOTOS: GALE ZUCKER

Q What is a CRA assessment and what does it do?

A CRA is a way for teachers to look at the models students use to solve problems. The students work on similar problems at three different stations in three different ways—concretely, representationally, and abstractly. It's primarily a screening activity, so you're looking for patterns in the class.

At the first station, students use physical materials—place-value blocks, Unifix cubes—to solve a problem. At the representational level, you're talking about drawings at the lower end to sophisticated models like charts, organized lists, diagrams, and graphs. Finally, at the abstract level, you're asking kids to work with equations to solve problems.

In America, particularly, we have an overemphasis on procedural understanding. Most teachers see the end goal as knowing the procedure instead of understanding the mathematical idea. So I like to tell teachers, "No model, no understanding."

Q How does a CRA give a better snapshot than traditional tests?

A A few years ago I was looking at kids' answers for the equation 17 + 17 + 17. A lot of them got 42. It turns out they were adding the three 7s in the ones column and got 21. They put down the 2 and carried the 1. When I asked them, "Why are you carrying the 1?" the response I got most often was, "Because that's what you do—you carry the 1."

In thinking about it mathematically, anytime you add two numbers together, you're never going to carry more than one. What they had taken from this idea of "carrying" was to "carry the 1," not to regroup into 10s. Knowing that dictated a different intervention path than some other kind of random answer. So "42" wasn't just a wrong answer. With a little bit more information, it gave me insight into the students' conceptual thinking about place value.

Q And isn't deeper conceptual understanding exactly what the Common Core calls for?

A Absolutely: "deeper, not wider." if you look at the practice standards, you realize that just the way CRA is set up is going to support most of them. There's a whole standard about modeling. You want students to be able to think of multiple ways to solve problems. But the real connection is the depth. CRA helps teachers look deeply into students' conceptualization, so they understand how students understand a mathematical concept.

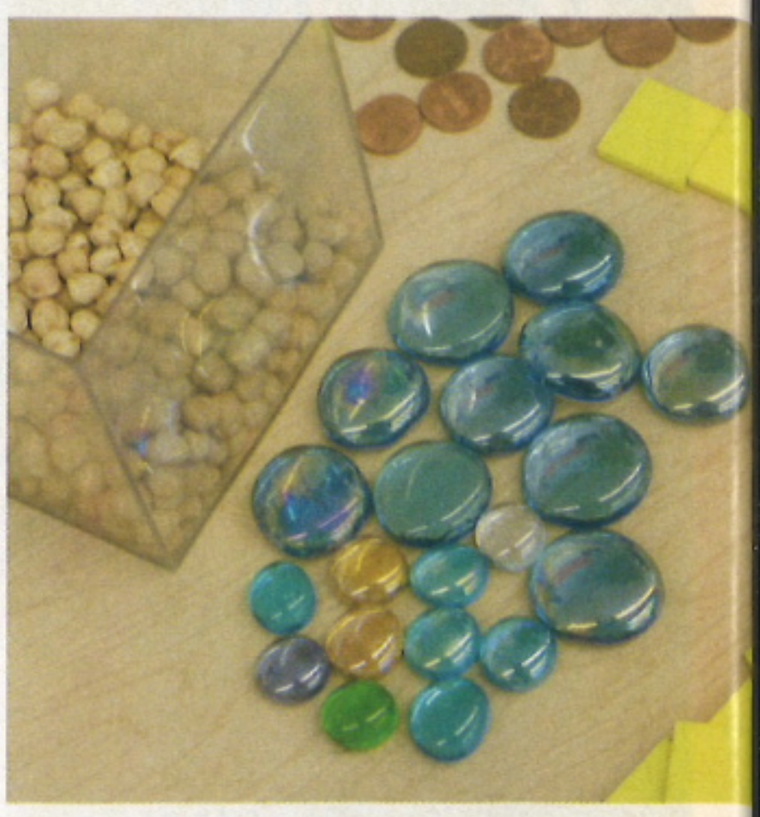
Getting a right answer on a test represents the potential for understanding, but it could be at the level of "I did the right steps—I followed the procedure." With fractions, for example, where there's heavy emphasis on procedure, it's not uncommon for a student to be able to write an equation to solve a problem but not be able to create any model at all. This is where instruction falls down. When you learn procedures, it's a trick you do for your math teacher.

And with students who've been identified as really having problems with math, teachers report to me that when they start an interview with them, almost to a teacher they go in thinking, "This child is on an IEP, and the reason they're struggling is because there's an attention deficit or they have a memory or processing issue." But once they start to look at the student's thinking, in every instance but one that I can remember, they find it's not a cognitive problem at all. It's a mathematical issue. They don't understand place value. Or they don't understand grouping. Or they don't really know what a fraction is.

On the one hand, it's a really exciting outcome because these things can be fixed. On the other hand, it's a little depressing that there's this rush to say, Well, this student has some cognitive challenge because they're unable to do fractions or decimals.

Q So we can go from "this kid has some problem learning" to identifying some very specific problem that can be addressed.

A It's pretty much accepted among teachers, particularly among literacy experts, that if you do certain things, virtually every kid will learn to read. I've taught kids to read in my career, and I've found that's pretty much the case. Every once in a while you'll get a kid who requires some specialized instruction because of a cognitive challenge. In general if you attend to certain things, reading's going to go well. In this way, math is really behind reading. In math, we don't know as much about the way mathematics develops, and we particularly don't know as much about teaching and interventions that support kids who



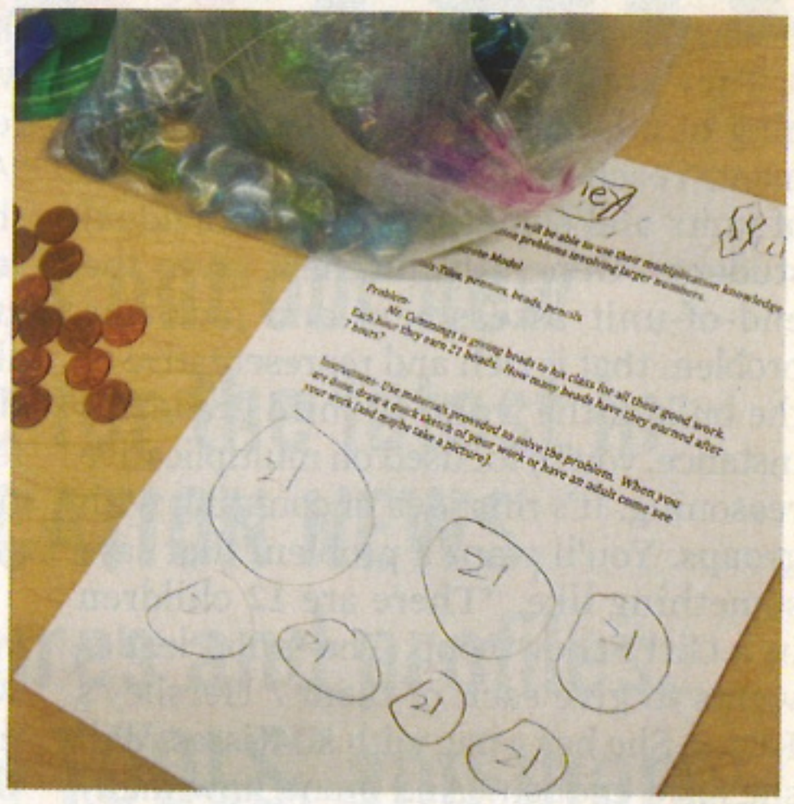
THE CONCRETE STATION

GROUP WORK

Fourth graders at the Academy School in Brattleboro, Vermont, use manipulatives at the concrete station to solve a multiplication word problem (essentially, 7 x 21).

The station draws out whether students are able to think in terms of grouping, a key concept at the heart of multiplication.

When they're done with their work, they draw a picture of it, or the teacher comes around and takes a photo.



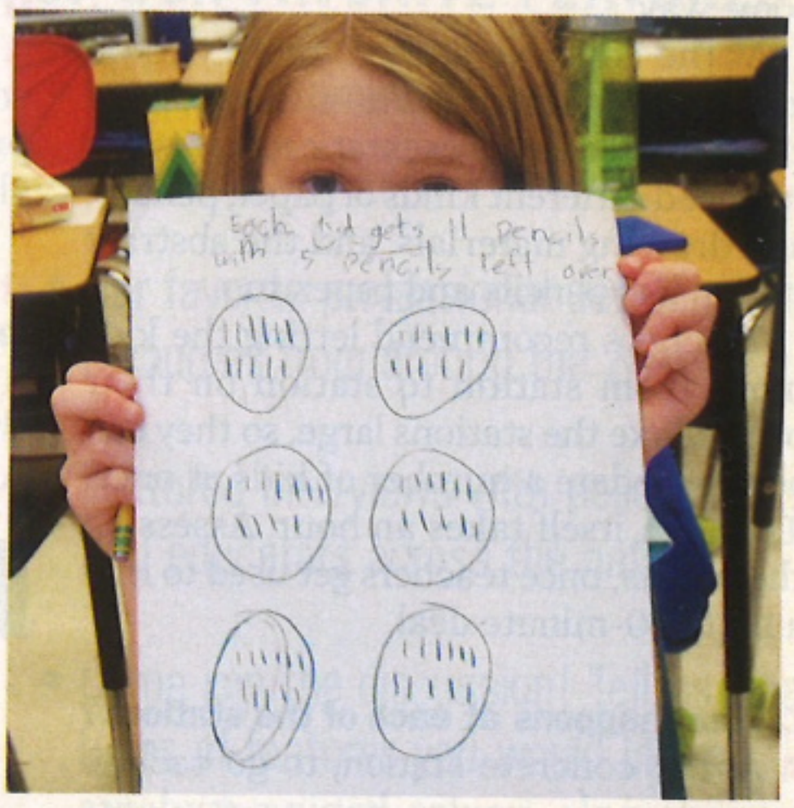
THE REPRESENTATIONAL STATION

DRAWING CONNECTIONS

Here, the students are equipped with various writing and drawing tools, graph paper, and unlined paper. They are instructed to use words and pictures to show their thinking. The word problem has been modified (this time, 71 ÷ 6).

Responses often use marks and circles to represent any number of methods for counting and grouping.

This station is "the really rich part," where kids make connections and develop a range of strategies for solving problems.



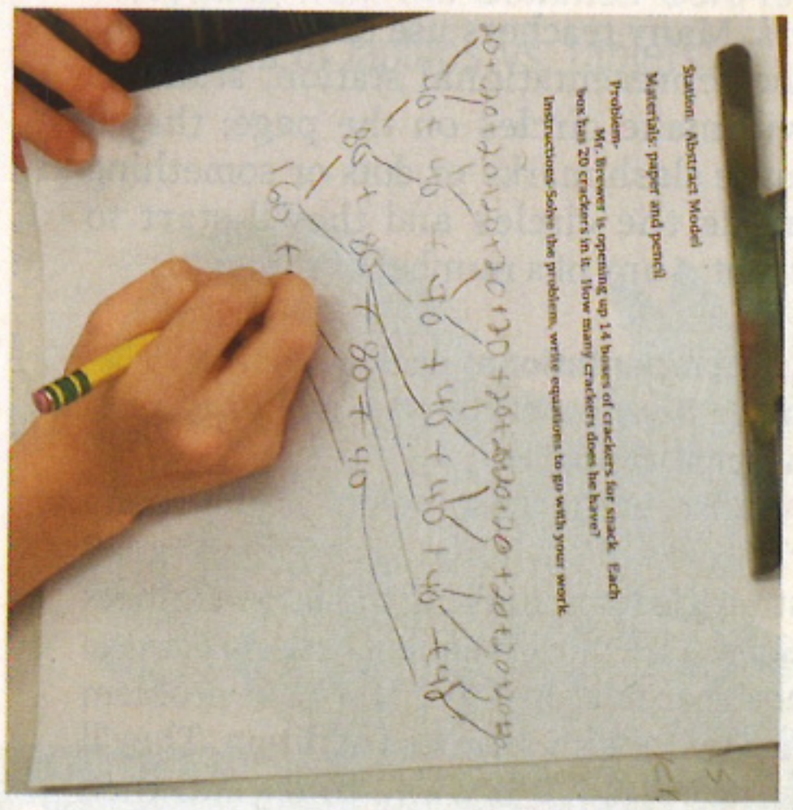
THE ABSTRACT STATION

NUMBERS GAME

Finally, students are asked to solve a similar problem (here, 14 x 20) using equations and with just paper and pencil.

Typically, students learning multiplication will represent the problem as an equation using repeated addition.

The work at this station also sheds light on the variety of strategies students employ to solve equations.



struggle. Some researchers say we're 15 years behind reading in this.

Q What do CRAs look like in action?
A They are usually given at the beginning of a large unit, as a needs assessment. Teachers also use them at the end of units and sometimes with individual students. Generally, teachers go to the end-of-unit assessment and pick one problem that is rich and representative of the bulk of the work. In third grade, for instance, you're focused on multiplicative reasoning. It's this idea of counting with groups. You'll want a problem that says something like, "There are 12 children in a Girl Scout troop. The troop leader wants to give each of them 7 Hershey's Kisses. She has a bag with 80 Kisses. Will she have enough? And if not, how many can everyone get?" Then you alter that slightly for each of the three stations, by changing the numbers or context in some way.

At the concrete station, there will be lots of materials available—tiles or Unifix cubes. At the representational station, you need different kinds of paper, pencils, and drawing materials, and the abstract station has pencils and paper, too.

Teachers recommend letting the kids move from station to station on their own. Make the stations large, so they can accommodate a number of kids at once. The CRA itself takes an hour. Assessing the results, once teachers get used to it, is a 15- to 20-minute deal.

Q What happens at each of the stations?
A At the concrete station, to go back to our example, you're hoping students make groups of seven with the objects and do some counting. The best way to record the concrete work is with a camera. Many teachers use their phones. At the representational station, students will make circles on the page; they'll make slash marks or dots or something inside the circles and they'll start to count ... any of a number of things.

Q Representational work seems to range from concrete-style markers to nascent formulas.
A Absolutely. The middle is the really rich part. It's where kids are developing strategies and making the large connections. The most frequent response at the abstract station for this sort of problem has to do with repeated addition. They'll write "7, 7, 7, 7" twelve times and then add them all together.

Q How do you assess the CRAs?
A The best process I've seen is for two teachers to do it together. They go through the pile and make comments on what they see: "Oh, a lot of kids are doing repeated addition in the abstract pile," or "A lot of kids use circles and slashes to show groupings," or "Kids at the concrete station are making long stacks of Unifix cubes—they're not grouping." That takes about five minutes. Then you go through the work a second time and sort by those features. Usually, you end up with four or five piles and have a good sense of what's going on with the class as a whole.

There are always pieces of work where you say, "I have no idea what this kid was thinking." It's a drawing you can't make sense of, or there are numbers on the page without any context. These go into the questions pile, also known as the "What the heck?" pile.

Q What do you do with those?
A That's where the flexible interview comes in. You sit down, present the same problem, and ask the student to think aloud.

Q How do teachers use what they've learned to inform instruction?
A Let's suppose I have five kids who made those ultralong stacks of Unifix cubes. And the other parts of their CRAs also indicate the notion of grouping is lacking. During the individualized, differentiated part of the lesson—"the menu," as I call it—I'll pull that group together, and I might play the game Circles and Stars with them. They roll the dice and make a certain number of circles. They roll the dice again and make that number of stars in each circle. Then they find the total. After playing the game for a couple of days, I ask, "Could you do this without actually drawing the stars?" and "How would you go about doing that?"

Q And CRA challenges teachers to think conceptually, too.
A It's used for professional development for just that reason. Teachers have to have the mathematical understanding. To recognize what it means for students to understand, for example, multiplicative reasoning deeply, teachers need to have that understanding as well. I think that's an area where there's some anxiety. I tell my students I know very few of them are becoming elementary school teachers because they want to teach math. They want to teach reading. But all of them are going to be math teachers, so they need to develop a deep knowledge of the math itself. One really common thing I hear is, "The second and third time I used CRA, I knew so much more."

Q I remember the joy of math was when something would click and I could visualize the concept.
A One of the worst parts of procedural-only is that all of the magic of math is lost. Why would you be interested? No disrespect to accountants, but I think of the procedural approach as accounting. They're algorithmic. A machine could do them. But that moment where you visualize the mathematics, that's the creation of the model. When you have that, you understand.

I have all these undergrads who are terrified of math. I even had a student cry in my office before class started—that's how uptight they are about it. But once they start to get a new look into math, several will find not only, "Oh wow, this is interesting," but "I'm good at this. I never knew I was good at this. People always told me I wasn't good at this, but if I think about it from this perspective I totally get it." And it's a wonderful thing. You think, Why did it take so long for them to get this information? ■

ADDITIONAL RESOURCES

CURRICULUM ASSOCIATES. The Ready Common Core package provides materials built to the new math standards from the ground up, along with an online tool kit. *curriculumassociates.com*

PEOPLES EDUCATION. You'll find free online videos that give an overview of the Common Core and dive deeper into the shifts at each grade level, as well as a set of CCSS workbooks, and more. *peopleseducation.com*

MATH SOLUTIONS. With its rich lineup of webinars, school-based coaching, and PD books (Tapper's *Solving for Why* among them), Math Solutions can ease the transition to the new standards. *mathsolutions.com*

MATH COMMON CORE COALITION. The NCTM has assembled a trove of Core-related curriculum, PD, and assessment resources. *mathccc.org*

Download Sample CRAs: *bit.ly/AdminCRA*